

# Equation of state for dense supernova matter

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April 2, 2009

## Abstract

We provide an equation of state for high density supernova matter by applying a momentum-dependent effective interaction. We focus on the study of the equation of state of high-density and high-temperature nuclear matter containing leptons (electrons and neutrinos) under the chemical equilibrium condition. The conditions of charge neutrality and equilibrium under  $\beta$ -decay process lead first to the evaluation of the lepton fractions and afterwards the evaluation of internal energy, pressure, entropy and in total to the equation of state of hot nuclear matter for various isothermal cases. Thermal effects on the properties and equation of state of nuclear matter are evaluated and analyzed in the framework of the proposed effective interaction model. Since supernova matter is characterized by a constant entropy we also present the thermodynamic properties for isentropic case. Special attention is dedicated to the study of the contribution of the components of  $\beta$ -stable nuclear matter to the entropy per particle, a quantity of great interest for the study of structure and collapse of supernova.

PACS number(s): 21.65.+f, 21.30.Fe, 24.10.Pa, 26.60.+c, 26.50.+x, 26.60.Kp

Keywords: Hot Nuclear Matter, Effective Interaction, Equation of State, Nuclear Symmetry Energy, Proton Fraction, Supernova, Neutron Star.

## 1 Introduction

Knowledge of the properties of the equation of state (EOS) of hot asymmetric nuclear matter is of fundamental importance to understand the physical mechanism of the iron core collapse of a massive star which produces a type-II supernova, and the rapid cooling of a new born hot neutron star. Additionally, the EOS defines the chemical composition, both qualitative and quantitative, of the hot nuclear matter. [1, 2, 3, 4]. Supernova explosions and neutron stars provide a unique laboratory where the EOS of nuclear matter can be investigated. A great opportunity to explore the EOS and properties of dense neutron-rich matter is available at the accelerator facility at GSI, and in future, will be also available through the high-energy radioactive beams at the planned Facility for Rare Isotope Accelerator (FRIA) [5, 6].

There is a wealth of existing literature regarding the EOS of supernova matter [1, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Supernova matter which exists in a collapsing supernova core and eventually forms a hot neutron star at birth is another form of nuclear matter distinguished in the participation of degenerate neutrinos and electrons [27]. It is characterized by almost constant entropy per baryon  $S = 1 - 2$  (in units

of the Boltzmann constant  $k_B$ ) throughout the density  $n$  and also by a high and almost constant lepton fraction  $Y_l = 0.3 - 0.4$  in contrast with ordinary neutron star matter where  $S = 0$  and  $Y_l \leq 0.05$ . These characteristics are caused by the effects of neutrino-trapping which occurs in the dense supernova core where a neutron star is formed.

This paper is a continuation of our previous work concerning the EOS of hot  $\beta$ -stable nuclear matter in cases where neutrinos have left the system [33]. More specifically, in order to study the properties and the EOS of hot nuclear matter, a momentum-dependent effective interaction model (MDIM) has been applied, one which is able to reproduce the results of more microscopic calculations of dense matter at zero temperature and which can be extended to finite temperature [2, 33, 34, 35]. The main incentive for the present study is the fact that only few calculations of the equation of state of the supernova matter at high densities are available, although at lower densities ( $n < n_0$ ) (where  $n_0 = 0.16 \text{ fm}^{-3}$  is the saturation density) reliable results are already available. For our purposes, we have applied a model which, in comparison to previous models, has the specific property that the temperature affects not only the kinetic, but also the interaction part of the energy density. In this way, we are able to simultaneously study thermal effects on the kinetic part of the symmetry energy and symmetry free energy, in addition to the interaction part of the above quantities [34]. This is significant in the sense that the density dependent behavior of the symmetry energy and symmetry free energy strongly influence the values of the proton fraction and as a consequence the composition of hot  $\beta$ -stable nuclear matter.

Our focus of interest is on the study of dense supernova matter. It has been speculated that matter at densities up to about  $n = 4n_0$  may be present in the core collapse of type-II supernova [1]. The present work can also be applied to the study of a neutron star at its birth, which is of particular interest as such a star creates a new form of matter under extreme conditions. In particular, proto-neutron stars are identified as a final stage of a supernova collapse. At this stage, a proto-neutron star is hot and composed of the so-called supernova matter.

In addition, we examine the two findings of the previous work of Takatsuka et al. [27, 36] concerning supernova matter. The first one is concerned with the finding that the population of the components is remarkably constant both in baryon density  $n$  and temperature  $T$  and the proton fraction  $Y_p$  is very large (e.g.,  $Y_p \simeq 0.3$  for  $Y_l = 0.4$ ) in contrast with that of neutron star matter. The second one concerns the finding that the EOS of dense supernova matter is by far stiffer than that of neutron star matter and correspondingly, hot neutron stars at birth are not only "fat" but hot as well compared to usual cold neutron stars. We broaden our study further by examining the influences of the temperature on the stiffness of EOS compared to the cold case.

The article is organized as follows. In section II the model and relative formulas are discussed and analyzed. Results are reported and discussed in section III, whereas the summary of the work is given in section IV.

## 2 The model

The model we use here, which has already been presented and analyzed in our previous papers [32, 33, 34, 35], is designed to reproduce the results of the microscopic calculations of both nuclear and neutron-rich matter at zero temperature and can be extended to finite temperature [2]. We provide the main characteristics of the model as follows:

The energy density of the asymmetric nuclear matter (ANM) is given by the relation

$$\epsilon(n_n, n_p, T) = \epsilon_{kin}^n(n_n, T) + \epsilon_{kin}^p(n_p, T) + V_{int}(n_n, n_p, T), \quad (1)$$

where  $n_n$  ( $n_p$ ) is the neutron (proton) density and the total baryon density is  $n = n_n + n_p$ . The

contribution of the kinetic parts are

$$\epsilon_{kin}^n(n_n, T) + \epsilon_{kin}^p(n_p, T) = 2 \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} (f_n(n_n, k, T) + f_p(n_p, k, T)), \quad (2)$$

where  $f_\tau$ , (for  $\tau = n, p$ ) is the Fermi-Dirac distribution function.

Including the effect of finite-range forces between nucleons, the potential contribution is parameterized as follows [2]

$$\begin{aligned} V_{int}(n_n, n_p, T) = & \frac{1}{3} A n_0 \left[ \frac{3}{2} - \left( \frac{1}{2} + x_0 \right) I^2 \right] u^2 + \frac{\frac{2}{3} B n_0 \left[ \frac{3}{2} - \left( \frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma+1}}{1 + \frac{2}{3} B' \left[ \frac{3}{2} - \left( \frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma-1}} \\ & + u \sum_{i=1,2} \left[ C_i \left( \mathcal{J}_n^i + \mathcal{J}_p^i \right) + \frac{(C_i - 8Z_i)}{5} I \left( \mathcal{J}_n^i - \mathcal{J}_p^i \right) \right], \end{aligned} \quad (3)$$

where

$$\mathcal{J}_\tau^i = 2 \int \frac{d^3k}{(2\pi)^3} g(k, \Lambda_i) f_\tau(n_\tau, k, T). \quad (4)$$

In Eq. (3),  $I$  is the asymmetry parameter ( $I = (n_n - n_p)/n$ ) and  $u = n/n_0$ , with  $n_0$  denoting the equilibrium symmetric nuclear matter density,  $n_0 = 0.16 \text{ fm}^{-3}$ . The asymmetry parameter  $I$  is related to the proton fraction  $Y_p$  by the equation  $I = (1 - 2Y_p)$ . The parameters  $A$ ,  $B$ ,  $\sigma$ ,  $C_1$ ,  $C_2$  and  $B'$  which appear in the description of symmetric nuclear matter are determined in order that  $E(n = n_0) - mc^2 = -16 \text{ MeV}$ ,  $n_0 = 0.16 \text{ fm}^{-3}$ , and the incompressibility are  $K = 240 \text{ MeV}$ . The additional parameters  $x_0$ ,  $x_3$ ,  $Z_1$ , and  $Z_2$  used to determine the properties of asymmetric nuclear matter are treated as parameters constrained by empirical knowledge [2].

The first two terms of the right-hand side of Eq. (3) arise from local contact nuclear interaction which lead to power density contributions as in the standard Skyrme equation of state. The first one corresponds to attractive interaction and the second one to repulsive. They are assumed to be temperature independent. The third term describes the effects of finite range interactions according to the choice of the function  $g(k, \Lambda_i)$ , and is the temperature dependent part of the interaction [5]. This interaction is attractive and important at low momentum, but it weakens and disappears at very high momentum. The function,  $g(k, \Lambda_i)$ , suitably chosen to simulate finite range effects, has the following form

$$g(k, \Lambda_i) = \left[ 1 + \left( \frac{k}{\Lambda_i} \right)^2 \right]^{-1}, \quad (5)$$

where the finite range parameters are  $\Lambda_1 = 1.5k_F^0$  and  $\Lambda_2 = 3k_F^0$  and  $k_F^0$  is the Fermi momentum at the saturation point  $n_0$ .

The energy density of asymmetric nuclear matter at density  $n$  and temperature  $T$ , in a good approximation, is expressed as

$$\epsilon(n, T, I) = \epsilon(n, T, I = 0) + \epsilon_{sym}(n, T, I), \quad (6)$$

where

$$\epsilon_{sym}(n, T, I) = nI^2 E_{sym}^{tot}(n, T) = nI^2 \left( E_{sym}^{kin}(n, T) + E_{sym}^{int}(n, T) \right). \quad (7)$$

In Eq. (7) the nuclear symmetry energy  $E_{sym}^{tot}(n, T)$  is separated into two parts corresponding to the kinetic contribution  $E_{sym}^{kin}(n, T)$  and the interaction contribution  $E_{sym}^{int}(n, T)$ .

From Eqs. (6) and (7) and setting  $I = 1$ , we find that the nuclear symmetry energy  $E_{sym}^{tot}(n, T)$  is given by

$$E_{sym}^{tot}(n, T) = \frac{1}{n} (\epsilon(n, T, I = 1) - \epsilon(n, T, I = 0)). \quad (8)$$

Thus, from Eq. (8) and by a suitable choice of the parameters  $x_0$ ,  $x_3$ ,  $Z_1$  and  $Z_2$ , we can obtain different forms for the density dependence of the symmetry energy  $E_{sym}^{tot}(n, T)$ .

It is well known that the need to explore different forms for  $E_{sym}^{tot}(n, T)$  stems from the uncertain behavior at high density [2]. The high-density behavior of symmetry energy is the worst known property of dense matter [37, 38, 39], with different nuclear models giving contradictory predictions. Recently, the density dependence of the symmetry energy in the equation of state of isospin asymmetric nuclear matter has been studied using isoscaling of the fragment yields and the antisymmetrized molecular dynamic calculation [40]. It was observed that the experimental data at low densities are consistent with the form of symmetry energy,  $E_{sym}(u) \approx 31.6u^{0.69}$ , in close agreement with those predicted by the results of variational many-body calculations. In Ref. [40] it was also suggested that the heavy ion studies favor a dependence of the form  $E_{sym}(u) \approx 31.6u^\gamma$ , where  $\gamma = 0.6 - 1.05$ . This constrains the form of the density dependence of the symmetry energy at higher densities, ruling out an extremely "stiff" and "soft" dependence [40].

In a previous study conducted with isospin dependent Boltzmann-Uehling-Uhlenbeck transport calculations, Chen et al. [41] have shown that a stiff density dependence of the symmetry energy parameterized as  $E_{sym}(u) \approx 31.6u^{1.05}$  clearly explains the isospin diffusion data [42] from NSCL-MSU (National Superconducting Cyclotron Laboratory at Michigan State University).

In the present work, since we are primarily interested in the study of thermal effects on the nuclear symmetry energy and free energy, we choose a specific form for this, enabling us to accurately reproduce the results of many other theoretical studies [43, 44]. In Ref. [43] the authors carried out a systematic analysis of the nuclear symmetry energy in the formalism of the relativistic Dirac-Brueckner-Hartree-Fock approach using the Bonn one-boson-exchange potential. In a very recent work, [44] the authors applied a similar method as in Ref. [43] for the microscopic predictions of the equation of state in asymmetric nuclear matter. In this case  $E_{sym}(u)$  is obtained with the simple parametrization  $E_{sym}(u) = Cu^\gamma$  with  $\gamma = 0.7 - 1.0$  and  $C \approx 32$  MeV. The authors concluded that a value of  $\gamma$  close to 0.8 gives a reasonable description of their predictions although the use of different functions in different density regions may be best for an optimal fit [44]. The results of Refs. [43, 44] are well reproduced by parameterizing the nuclear symmetry energy according to the following formula

$$E_{sym}^{tot}(n, T = 0) = 13u^{2/3} + 17F(u), \quad (9)$$

where the first term of the right part of Eq. (9) corresponds to the contribution of the kinetic energy and the second term to the contribution of the interaction energy.

For the function  $F(u)$ , which parameterizes the interaction part of the symmetry energy, we apply the following form

$$F(u) = u. \quad (10)$$

The parameters  $x_0$ ,  $x_3$ ,  $Z_1$  and  $Z_2$  are chosen so that Eq. (8), for  $T = 0$  reproduces the results of Eq. (9) for the function  $F(u) = u$ .

In general, in order to obtain different forms for the density dependence of  $E_{sym}(n)$ , the function  $F(u)$  can be parameterized as follows [2]

$$F(u) = \sqrt{u}, \quad F(u) = u, \quad F(u) = 2u^2/(1 + u). \quad (11)$$

It is worthwhile to point out that the above parametrization of the interacting part of the nuclear symmetry energy is extensively used for the study of neutron star properties [2, 45] as well as the study of the collisions of neutron-rich heavy ions at intermediate energies [46, 47]. For a

very recent review of the applications of the proposed momentum dependent effective interaction model and the specific parametrization of it see Ref. [48] (and references therein).

## 2.1 Thermodynamic description of hot nuclear matter

In order to study the properties of nuclear matter at finite temperature, we need to introduce the Helmholtz free energy  $F$  which is written as [49, 50]

$$F(n, T, I) = E(n, T, I) - TS(n, T, I). \quad (12)$$

In Eq. (12),  $E$  is the internal energy per particle,  $E = \epsilon/n$ , and  $S$  is the entropy per particle,  $S = s/n$ . From Eq. (12) it is also concluded that for  $T = 0$ , the free energy  $F$  and the internal energy  $E$  coincide.

The entropy density  $s$  has the same functional form as that of a non-interacting gas system, given by the equation

$$s_\tau(n, T, I) = -2 \int \frac{d^3k}{(2\pi)^3} [f_\tau \ln f_\tau + (1 - f_\tau) \ln(1 - f_\tau)], \quad (13)$$

while the pressure and the chemical potentials defined as follows [49, 50]

$$P = n^2 \left( \frac{\partial \epsilon/n}{\partial n} \right)_{S, N_i}, \quad \mu_i = \left( \frac{\partial \epsilon}{\partial n_i} \right)_{S, V, n_{j \neq i}}. \quad (14)$$

At this point we shall examine the properties and the EOS of nuclear matter by considering an isothermal process. In this case, the pressure and the chemical potentials are related to the derivative of the total free energy density  $f = F/n$ . More specifically, they are defined as follows

$$P = n^2 \left( \frac{\partial f/n}{\partial n} \right)_{T, N_i}, \quad \mu_i = \left( \frac{\partial f}{\partial n_i} \right)_{T, V, n_{j \neq i}}. \quad (15)$$

The pressure  $P$  can also be calculated from the equation [49, 50]

$$P = Ts - \epsilon + \sum_i \mu_i n_i. \quad (16)$$

It is also possible to calculate the entropy per particle  $S(n, T)$  by differentiating the free energy density  $f$  with respect to the temperature, that is

$$S(n, T) = - \left( \frac{\partial f/n}{\partial T} \right)_{V, N_i}. \quad (17)$$

The comparison of the two entropies, that is from Eqs. (13) and (17), provides a test of the approximation used in the present work (see Ref. [33]). It is easy to demonstrate by applying Eq. (15) that (see for a proof [45] as well as [51])

$$\begin{aligned} \mu_n &= F + u \left( \frac{\partial F}{\partial u} \right)_{Y_p, T} - Y_p \left( \frac{\partial F}{\partial Y_p} \right)_{n, T}, \\ \mu_p &= \mu_n + \left( \frac{\partial F}{\partial Y_p} \right)_{n, T}, \\ \hat{\mu} &= \mu_n - \mu_p = - \left( \frac{\partial F}{\partial Y_p} \right)_{n, T}. \end{aligned} \quad (18)$$

We can define the symmetry free energy per particle  $F_{sym}(n, T)$  by the following parabolic approximation (see also [51, 52])

$$F(n, T, I) = F(n, T, I = 0) + I^2 F_{sym}(n, T) = F(n, T, I = 0) + (1 - 2Y_p)^2 F_{sym}(n, T), \quad (19)$$

where

$$F_{sym}(n, T) = F(n, T, I = 1) - F(n, T, I = 0). \quad (20)$$

It is worth noting that the above approximation is not valid from the beginning, but one needs to check the validity of the parabolic law in the present model before using it. In Ref. [33] we have proved the validity of the approximation (19).

Now, by applying Eq. (19) in Eq. (18), we obtain the key relation

$$\hat{\mu} = \mu_n - \mu_p = 4(1 - 2Y_p)F_{sym}(n, T). \quad (21)$$

The above equation is similar to that obtained for cold nuclear matter by replacing  $E_{sym}(n)$  with  $F_{sym}(n, T)$ . The finding that, for both quantities ( $E(n, T, I)$  and  $F(n, T, I)$ ), the dependence of the asymmetry parameter  $I$  can be approximated very well by a quadratic dependence leads to the conclusion that the entropy  $S(n, T, I)$  must also exhibit quadratic dependence of  $I$  that is

$$S(n, T, I) = S(n, T, I = 0) + I^2 S_{sym}(n, T) \quad (22)$$

where

$$S_{sym}(n, T) = S(n, T, I = 1) - S(n, T, I = 0) = \frac{1}{T}(E_{sym}(n, T) - F_{sym}(n, T)). \quad (23)$$

In order to check the validity of the parabolic approximation (23), we plot in Fig. 1 the difference  $S(n, T, I = 1) - S(n, T, I = 0)$  as a function of  $I^2$  at temperature  $T = 10$  and  $T = 30$  MeV for two baryon densities, i.e.,  $n = 0.2 \text{ fm}^{-3}$  and  $n = 0.4 \text{ fm}^{-3}$ . It is thus evident that in a good approximation, an almost linear relation holds between  $S(n, T, I = 1) - S(n, T, I = 0)$  and  $I^2$ , especially for low values of  $I^2$ .

Also noteworthy in the present model is that due to temperature dependence of the interaction part of the energy density, the temperature affects both the kinetic part contribution on the entropy  $S$  and the interaction part. Hence, the total entropy per baryon must be written as follow  $S_{tot} = S_{kin} + S_{int}$ .

## 2.2 $\beta$ -equilibrium in hot proto-neutron star and supernova

Stable high density nuclear matter must be in chemical equilibrium with all type of reactions, including the weak interactions in which  $\beta$  decay and electron capture take place simultaneously

$$n \longrightarrow p + e^- + \bar{\nu}_e, \quad p + e^- \longrightarrow n + \nu_e. \quad (24)$$

Both types of reactions change the electron per nucleon fraction,  $Y_e$  and thus affect the equation of state. In a previous study, we assumed that neutrinos generated in these reactions left the system [33]. The absence of neutrino-trapping has a dramatic effect on the equation of state and is the main cause of a significant reduction in the values of the proton fraction  $Y_p$  [27, 36].

The chemical equilibrium of reactions (24) can be expressed in terms of the chemical potentials for the four species

$$\mu_n + \mu_{\nu_e} = \mu_p + \mu_e. \quad (25)$$

The charge neutrality condition provides us with the equation

$$Y_p = Y_e, \quad (26)$$

while the total fraction of leptons is given by

$$Y_l = Y_e + Y_{\nu_e}. \quad (27)$$

Now, according to Eqs. (21) and (25) we have

$$\mu_e - \mu_{\nu_e} = \mu_n - \mu_p = 4(1 - 2Y_p) F_{sym}(n, T). \quad (28)$$

Moreover, the leptons (electrons, muons and neutrinos) density is given by the expression

$$n_l = \frac{g}{(2\pi)^3} \int \frac{d\mathbf{k}}{1 + \exp \left[ \frac{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4} - \mu_l}{T} \right]}, \quad (29)$$

where  $g$  stands for the spin degeneracy ( $g = 2$  for electrons and muons and  $g = 1$  for neutrinos). One can self-consistently solve Eqs. (26), (27), (28) and (29) in order to calculate the proton fraction  $Y_p (= Y_e)$ , the neutrino fractions  $Y_{\nu_e}$ , as well as the electron chemical potential  $\mu_e$  as a function of the baryon density  $n$  for various values of temperature  $T$ .

Depending on the form of the symmetry energy, muons can appear at high density. Prakash [45] has shown that the more rapidly  $F(u)$  increases with density, the lower the density at which muons appear. For example, with  $F(u) = u$ , muons appear at  $u \sim 4$ , while with the choice  $F(u) = \sqrt{u}$ , muons cannot appear till  $u \sim 8$ . However, the presence of muons has very little effect on the electron lepton fractions (compared to the case without the inclusion of muons) since  $Y_\mu$  remains extremely small ( $\sim 10^{-4}$ ) over a wide range of densities [45]. Thus, we do not include the contribution of muons in our study.

The next step is to calculate the energy density and pressure of leptons given by the following formulae

$$\epsilon_l(n_l, T) = \frac{g}{(2\pi)^3} \int \frac{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4} d\mathbf{k}}{1 + \exp \left[ \frac{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4} - \mu_l}{T} \right]}, \quad (30)$$

$$P_l(n_l, T) = \frac{1}{3} \frac{g(\hbar c)^2}{(2\pi)^3} \int \frac{1}{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4}} \frac{k^2 d\mathbf{k}}{1 + \exp \left[ \frac{\sqrt{\hbar^2 k^2 c^2 + m_l^2 c^4} - \mu_l}{T} \right]}. \quad (31)$$

The entropy density  $s$  has the same functional form as that of a non-interacting gas system, given by the equation

$$s_l(n, T, I) = -g \int \frac{d^3 k}{(2\pi)^3} [f_l \ln f_l + (1 - f_l) \ln(1 - f_l)]. \quad (32)$$

The equation of state of hot nuclear matter in  $\beta$ -equilibrium (considering that it consists of neutrons, protons, electrons and neutrinos) can be obtained by calculating the total energy density  $\epsilon_{tot}$  as well as the total pressure  $P_{tot}$ . The total energy density is given by

$$\epsilon_{tot}(n, T, I) = \epsilon_b(n, T, I) + \sum_{l=e, \nu_e} \epsilon_l(n, T, I), \quad (33)$$

where  $\epsilon_b(n, T, I)$  and  $\epsilon_l(n, T, I)$  are the contributions of baryons and leptons respectively. The total pressure is

$$P_{tot}(n, T, I) = P_b(n, T, I) + \sum_{l=e, \nu_e} P_l(n, T, I), \quad (34)$$

where  $P_b(n, T, I)$  is the contribution of the baryons (see Eq. (16)) i.e.

$$P_b(n, T, I) = T \sum_{\tau=p,n} s_\tau(n, T, I) + \sum_{\tau=n,p} n_\tau \mu_\tau(n, T, I) - \epsilon_b(n, T, I), \quad (35)$$

while  $P_l(n, T, I)$  is the contribution of the leptons (see Eq. (31)). From Eqs. (33) and (34) we can construct the isothermal curves for energy and pressure and finally derive the isothermal behavior of the equation of state of hot nuclear matter under  $\beta$ -equilibrium.

### 3 Results and Discussions

We calculate the equation of state of hot asymmetric nuclear matter by applying a momentum dependent effective interaction model describing the baryons interaction. We consider that nuclear matter contains neutrons, protons, electrons and neutrinos under  $\beta$ -equilibrium and charge neutrality. The key quantities in our calculations are the proton fraction  $Y_p$  and also the asymmetry free energy defined in Eq. (20). It is worth pointing out that since the supernova explosion itself is a dynamic phenomenon, the chemical composition of matter changes according to the evolution of the star all the time [28]. During supernova explosion, the chemical composition of matter reaches equilibrium not in the whole star but locally. In our present work we assume matter in the chemical equilibrium for simplicity in order to analyze the properties of hot neutron star and supernova matter.

The validity of the parabolic approximation (19) has already been tested and presented in our previous work [33].  $F_{sym}(u, T)$ , for various values of the temperature  $T$ , was derived with a least-squares fit to the numerical values according to Eq. (20) and has the form [33]

$$\begin{aligned} F_{sym}(u; T=0) &= 13u^{2/3} + 17u \\ F_{sym}(u; T=5) &= 3.653 + 28.018u - 1.5126u^2 + 0.185u^3 - 0.010u^4, \\ F_{sym}(u; T=10) &= 5.995 + 26.157u - 0.827u^2 + 0.068u^3 - 0.002u^4, \\ F_{sym}(u; T=20) &= 13.200 + 21.267u + 0.800u^2 - 0.193u^3 + 0.014u^4, \\ F_{sym}(u; T=30) &= 21.087 + 17.626u + 1.645u^2 - 0.289u^3 + 0.018u^4. \end{aligned} \quad (36)$$

where the case with  $T=0$ , is included as well. In that case  $F_{sym}$  coincides with  $E_{sym}$ .

Firstly, in order to clarify the contribution of the three terms of the potential energy density, we plot the terms as a function of the baryon density, in Fig. 2(a). In that figure we have that

$$\begin{aligned} V^A &= \frac{1}{3} A n_0 \left[ \frac{3}{2} - \left( \frac{1}{2} + x_0 \right) I^2 \right] u^2, \\ V^B &= \frac{\frac{2}{3} B n_0 \left[ \frac{3}{2} - \left( \frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma+1}}{1 + \frac{2}{3} B' \left[ \frac{3}{2} - \left( \frac{1}{2} + x_3 \right) I^2 \right] u^{\sigma-1}}, \\ V^C &= u \sum_{i=1,2} \left[ C_i \left( \mathcal{J}_n^i + \mathcal{J}_p^i \right) + \frac{(C_i - 8Z_i)}{5} I \left( \mathcal{J}_n^i - \mathcal{J}_p^i \right) \right]. \end{aligned} \quad (37)$$

The first term  $V^A$  corresponds to attractive interaction where the second  $V^B$  corresponds to repulsive interaction and dominates for high values of  $n$  ( $n > 0.6 \text{ fm}^{-3}$ ). Both of the above terms are temperature independent. The third term  $V^C$  contains the momentum dependent part of the interaction, corresponds to attractive interaction and its main contribution is to compete the repulsive interaction of  $V^B$  for high values of  $n$  and as a consequence avoid acausal behavior of the EOS at high densities. The term  $V^C$  consists of two finite range terms, one corresponding to a long-range attraction and the other to a short-range repulsion.



Thermal effects on the momentum dependent term  $V^C$  are displayed in Fig. 2(b). The contribution of  $V^C$  is plotted for various values of  $T$ . It is concluded that thermal effects are more pronounced for high values of  $T$  ( $T > 10$  MeV) leading to a less attractive contribution. More precisely, we find that for small values of  $n$  (i.e.  $n = 0.15 \text{ fm}^{-3}$ )  $V^C$  increases (compared to the cold case  $T = 0$ ) 3% – 20% for  $T = 10 - 30$ . For higher values of  $n$  the increase is even less.

The outline of our calculations procedure approach is the following: For a fixed value of baryon density  $n$ , lepton fraction  $Y_l$ , temperature  $T$  and initial trial value of proton fraction  $Y_p$  ( $= Y_e$ ), Eq. (29) is solved in order to calculate the chemical potential  $\mu_e$ . The knowledge of  $\mu_e$  allows the calculation of  $\mu_{\nu_e}$  from Eq. (28) which may then be used to infer the neutrino fraction  $Y_{\nu_e}$  from Eq. (29). Finally, a new value of proton fraction  $Y_p$  ( $= Y_e$ ) is taken from equation  $Y_e = Y_l - Y_{\nu_e}$  and the procedure is repeated from the beginning until a convergence is achieved.

In Fig. 3 we plot the fraction of electrons  $Y_e$  and neutrinos  $Y_{\nu_e}$  versus the baryon density  $n$  for lepton fraction  $Y_l = 0.3$  and  $Y_l = 0.4$  and for various values of  $T$ . It is concluded that thermal effects are important, both for electron and neutrinos fractions for low values of the baryon density  $n$  i.e.  $n < 0.4 \text{ fm}^{-3}$ .  $Y_e$  is an increasing function of  $T$  and consequently  $Y_{\nu_e}$  is a decreasing function of  $T$ . For higher values of  $n$ , the thermal effects on lepton's fraction are unimportant.

At this point, following the discussion of Takatsuka et al. [27], we attempt to extend the discussion concerning the dependence of equilibrium fraction  $Y_e (= Y_p)$  on the baryon density as well as on the nuclear symmetry energy. We ignore the temperature effect to clarify the situation. Actually, the situation does not change by including finite temperature effects. The energy per baryon of supernova matter  $E_{SM}$  and cold neutron star matter  $E_{NS}$  are expressed as function of  $n$  and  $Y_p$  (see also ref. [27]) as

$$E_{SM}(n, Y_p) = E_b(n, Y_p) + E_e(n, Y_p) + E_{\nu_e}(n, Y_p) \quad (38)$$

$$= E_b(n, Y_p = 0.5) + E_{sym}(n)(1 - 2Y_p)^2 + 253.6u^{1/3}Y_p^{4/3} + 319.516u^{1/3}(Y_l - Y_p)^{4/3}$$

$$E_{NS}(n, Y_p) = E_b(n, Y_p) + E_e(n, Y_p) \quad (39)$$

$$= E_b(n, Y_p = 0.5) + E_{sym}(n)(1 - 2Y_p)^2 + 253.6u^{1/3}Y_p^{4/3}$$

where the symmetry energy  $E_{sym}(n)$  is parameterized according to Eq. (11).  $E_{sym}(n)$  is plotted in Fig. 4(a) for the three different parametrizations. In the same figure we have included recent results provided in reference [44] achieved by performing microscopic calculations in asymmetric nuclear matter. In this case  $E_{sym}(n)$  is obtained with the simple parametrization

$$E_{sym}(u) = Cu^\gamma$$

with  $\gamma = 0.8$  and  $C = 32$  MeV. It is obvious that the results of the above parametrization, correspond very well with the parametrization  $F(u) = u$  which is proposed here.

The equilibrium proton fraction  $Y_p$  is calculated by solving the equation  $\partial E_{SM,NS}/\partial Y_p = 0$  for various values of the density  $n$ ,  $E_{sym}(n)$  and  $Y_l = 0.4$  for supernova matter. The results are presented in Fig. 4(b). In the case of cold neutron star matter,  $Y_p$  depends strongly on both the baryon density and the values of the  $E_{sym}(n)$ . This is not the case for supernova matter where the effect of nuclear symmetry energy in determining  $Y_p$  is less important than in cold neutron star matter. In addition,  $Y_p$ , for a fixed parametrization of  $F(u)$  is almost constant with respect to  $n$ .

Fig. 5 displays thermal effects on the chemical potential of leptons for  $Y_l = 0.3$  and  $Y_l = 0.4$ . In all cases,  $\mu_l$  is a slightly decreasing function of  $T$ . In fact, the important quantity for our calculations is the difference  $\hat{\mu} = \mu_e - \mu_{\nu_e}$  which is strictly constrained from Eq. (28). So, it is appropriate to check the validity of Eq. (19) at least for proton fraction  $Y_p \approx 0.3$  (or  $I^2 \approx 0.16$ ). We found in our previous work [33] (Fig. 1), that Eq. (19) is accurate for the values of the proton

fraction which are under consideration in the present work. The use of the formula (28) is very useful since it can greatly simplify the coupled equations used for the construction of the EOS. We mention here that to our knowledge, the above treatment has never been applied for the study of the EOS of supernova matter and has been applied for the first time in the present work.

In Fig. 6 we plot the contribution of the baryons  $S_b$ , leptons  $S_l$  and the total  $S_{tot}$  to the entropy per baryon. In all cases,  $S$  is a decreasing function of the baryon density  $n$ . Temperature affects appreciably both baryon and lepton contribution. It should be noted that the contribution of baryons  $S_b$  may be written as  $S_b = S_{kin} + S_{int}$ , where the term  $S_{kin}$  originates from the temperature effect on the kinetic part of the energy density and  $S_{int}$  reflects thermal effects on the potential energy density. More precisely, entropy density  $s$ , according to equation (13), is an increasing function of the diffuseness of the Fermi-Dirac distribution  $f_\tau(n, k, T)$ . As indicated in our previous work [33] (Fig. 11), the effect of the diffuseness of the distribution  $f_\tau(n, k, T)$  is pronounced for low values of the baryon density  $n$  and for high values of  $T$ . But as we have shown above, thermal effect on the momentum dependent term  $V^C$  is important for low values of  $n$  and also high values of  $T$ . Therefore, we conclude that the term  $V^C$  has a more pronounced effect on the entropy density  $s$  mainly for low values of  $n$ . For higher values of  $n$  the contribution of  $V^C$  on  $s$  is less important.

Furthermore, the lepton contribution  $S_l$  is an increasing function of the lepton fraction  $Y_l$ , while the baryon contribution is almost independent by  $Y_l$ . For the electron and neutrino entropy density, our present results can be reproduced with good accuracy, at least for low values of  $T$ , by applying the analytical formula used by Onsi *et al.* [29, 30]

$$s_e = \frac{1}{3} \frac{\mu_e^2}{(\hbar c)^3} T, \quad \mu_e = \hbar c (3\pi^2 Y_e n)^{1/3}. \quad (40)$$

$$s_{\nu_e} = \frac{1}{6} \frac{\mu_{\nu_e}^2}{(\hbar c)^3} T, \quad \mu_{\nu_e} = \hbar c (6\pi^2 Y_{\nu_e} n)^{1/3}. \quad (41)$$

According to the above formula, the specific contribution of the leptons (electrons and neutrinos) to the entropy per baryon has the form

$$S_{e,\nu_e} = s_{e,\nu_e}/n \sim \left( \frac{Y_{e,\nu_e}^2}{n} \right)^{1/3} T. \quad (42)$$

Eq. (42) gives an nice explanation for the density and temperature dependence of  $S_l$  presented in Fig. 6(b).

In Fig. 7, we display the contribution to internal energy  $E$  from baryons  $E_b$  and leptons  $E_l$  for  $Y_l = 0.3$  and  $Y_l = 0.4$  and for various values of  $T$ . The most striking aspect is that the lepton energy,  $E_l = E_e + E_{\nu_e}$ , dominates in the internal energy of the matter up to  $n \sim 0.6 \text{ fm}^{-3}$  (for  $Y_l = 0.3$ ) and  $n \sim 0.8 \text{ fm}^{-3}$  (for  $Y_l = 0.4$ ). This is a characteristic of the supernova matter and is in remarkable contrast with the situation of cold neutron star matter [27]. The contribution from baryon  $E_b$  gets larger with the increase of  $n$  and is comparable with  $E_l$  for high values of  $n$ .

It is worth pointing out that the above situation depends on the combination of the stiffness of nuclear equation of state (values of incompressibility and density dependent behavior of the nuclear symmetry energy) and the lepton fraction. Nonetheless, the main feature is unaltered, especially up to low values of  $n$  [27]. Moreover, the contribution on the lepton energy, as is presented in Fig. 8, originates mainly from electrons while neutrino contribution is smaller (but not negligible).

The present results for the electron and neutrino energy per baryon, can also be accurately reproduced, at least for low values of  $T$ , by applying the analytical formula used by Onsi *et al.*

[29, 30] where the energy density  $\epsilon_l$  and energy per baryon  $E_l$  of the leptons are given by

$$\epsilon_e = \frac{1}{4\pi^2} \frac{\mu_e^4}{(\hbar c)^3} \left( 1 + \frac{2}{3} \frac{\pi^2 T^2}{\mu_e^2} \right), \quad E_e \sim (Y_e^4 n)^{1/3} \left( 1 + \mathcal{C} \frac{T^2}{(Y_e n)^{2/3}} \right) \quad (43)$$

$$\epsilon_{\nu_e} = \frac{1}{8\pi^2} \frac{\mu_{\nu_e}^4}{(\hbar c)^3} \left( 1 + \frac{2}{3} \frac{\pi^2 T^2}{\mu_{\nu_e}^2} \right), \quad E_{\nu_e} \sim (Y_{\nu_e}^4 n)^{1/3} \left( 1 + \tilde{\mathcal{C}} \frac{T^2}{(Y_{\nu_e} n)^{2/3}} \right). \quad (44)$$

In Eqs. (43) and (44),  $\mathcal{C}$  and  $\tilde{\mathcal{C}}$  are constants.

The contributions of baryon and leptons on the total pressure are presented in Fig. 9. In contrast to the situation of the internal energy, the nuclear part contribution plays a more important role compared with the lepton part. The lepton pressure  $P_l$  is comparable to baryon pressure  $P_b$  up to  $n \sim 0.2 \text{ fm}^{-3}$ , but for higher values of  $n$  it is significantly small. What is more, the main part of  $P_l$  originates from electrons compared to neutrinos as presented in Fig. 10.

As pointed out by Bethe et al. [9], the crucial feature in determining the evaluation of a collapsing pre-supernova core is that the entropy per particle is very low, of the order of unity (in units of the Boltzmann constant  $k_B$ ), and nearly constant during all the stages of the collapse up to the shock wave formation. Therefore, the collapse is an adiabatic process of a highly ordered system. So, since the supernova matter is characterized by a constant entropy and constant lepton fraction, we shall also discuss the properties under this condition. This can be done by converting the results for isothermal case ( $T=\text{const}$ ) into those for adiabatic case ( $S=\text{const}$ ) in terms of the  $T - n$  relation constrained by a constant entropy.

The  $T = T(n)$  relation is constructed by  $\{T, n\}$  values to satisfy  $S(n, T)=\text{const}$  in an  $S - n$  diagram. Fig. 11 shows the results for  $Y_l = 0.3$  and  $Y_l = 0.4$  for  $S = 1$ . Temperature is an increasing function of  $n$ . Furthermore, for the same density, the temperature is higher for lower values of  $Y_l$ . The values of  $T$  for various values of  $n$  are derived, for the two cases, with the least-squares fit method and found to take the general form

$$T(n) = an^b,$$

where  $a = 35.412$ ,  $b = 0.70481$  for  $Y_l = 0.3$  and  $a = 32.35706$ ,  $b = 0.67694$  for  $Y_l = 0.4$ . The results of this study are in very good agreement with those of Takatsuka et al. [27]. The stars at lower density denote the  $\{T, n\}$  values for  $S = 1$  and  $Y_l = 0.4$  which are derived from Lattimer et al. [19]. It is concluded that the temperature increases considerably when moving from the outer part of the star to the center in order to maintain a constant value of the entropy per baryon.

By applying the relation  $T - n$  presented in Fig. 11, the fractions  $Y_i$  of isothermal case (see Fig. 3) is converted into the isentropic one for  $S = 1$ . The population of constituents is plotted in Fig. 12 as a function of  $n$  for  $Y_l = 0.3$  and  $Y_l = 0.4$ . The most striking feature of the results is the slight dependence of the fraction  $Y_i$  from the baryon density  $n$  (the same behavior and similar results have been found in Ref. [27]). To sum up, during the adiabatic collapse of a supernova, the population of the constituents (neutrons, protons, electrons and neutrinos) are almost the same independent of the density  $n$ .

The entropy contributions (for  $S = 1$ ) from the constituents are presented in Fig. 13. The contributions of the baryon are increasing functions of  $n$ , the contribution of electrons is increasing function of  $n$ , while  $S_{\nu_e}$  is almost independent of density. Roughly, neutrons, protons, electrons and neutrinos contribute to  $S$  ( $= 1$ ) by about 50, 30, 17, 3% (for  $Y_l = 0.3$ ) and 45, 30, 20, 5% (for  $Y_l = 0.4$ ) respectively. Our results are quite consistent with those of Ref. [27].

Fig. 14, displays the internal energies per baryon of respective components  $E_i$  versus  $n$  for  $Y_l = 0.3$ ,  $Y_l = 0.4$  and  $S = 1$ . The main conclusions of the results are similar to those of the isothermal case (see Fig. 7). The lepton energy (which mainly originated from electrons

contributions) dominates in the internal energy of the supernova matter even for high values of the density  $n$ . The nuclear contribution on the internal energy dominates only in high values of  $n$  (depending on the lepton fraction  $Y_l$ ).

Finally, in Fig. 15 we compare the EOS's between supernova matter and cold neutron star matter. The case for supernova matter corresponds to  $S = 1$  and  $Y_l = 0.3$ . It is thus clear that the internal energy  $E_{tot}$  of supernova matter (SM) is remarkably larger than that of neutron star matter (NS). As far as the nucleon part  $E_b$  is concerned, the  $E_b$  in SM is slightly lower than that in NS due to the large energy gain in symmetry energy (see also [27]). However, the lepton contribution on the internal energy  $E_l$  is remarkably larger in SN matter compared to NS matter due to the effect of a large lepton fraction, that is, a large kinetic energy of abundant leptons. High temperature also contributes to the stiffening, but it is less effective than the high lepton fractions (see also Fig. 7). The present results also correspond well with those presented by Takatsuka et al. [27, 36] a few years ago

## 4 Summary

The evaluation of the equation of state of hot nuclear matter is a major challenge for nuclear physics and astrophysics. EOS is the basic ingredient necessary for studying the supernova explosion as well as for determining the properties of hot neutron stars. The motive for the present work has been to apply a momentum-dependent interaction model for the study of the hot nuclear matter EOS under  $\beta$ -equilibrium. We have calculated the lepton fractions by applying the constraints for chemical equilibrium and charge neutrality. The internal energy and also the pressure have been calculated as functions of baryon density and for various values of temperature. Special attention has been dedicated to the study of the contribution of the components of  $\beta$ -stable nuclear matter on the entropy per particle, a quantity of great interest in the study of structure and collapse of supernova. We have presented and analyzed the contribution of each component. Finally, we have presented the EOS of  $\beta$ -stable hot nuclear matter, by taking into account and analyzing the contributions to the total pressure of each component. The above EOS can be applied to the evaluation of the gross properties of hot neutron stars i.e. mass and radius.

## Acknowledgments

The author would like to thank Professor Tatsuyauki Takatsuka for valuable comments and correspondence.

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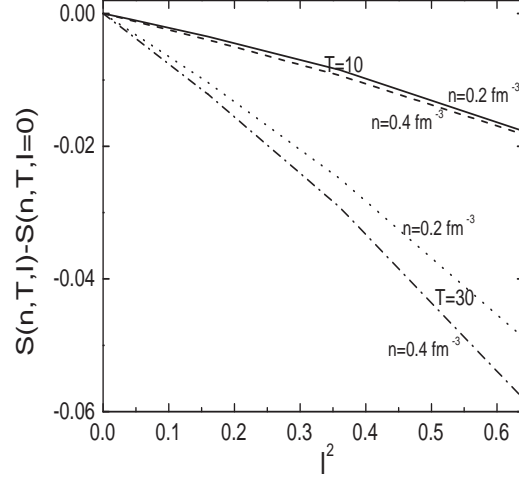


Figure 1: The difference  $S(n, T, I) - S(n, T, I = 0)$  as a function of  $I^2$  at temperature  $T = 10$  MeV and  $T = 30$  MeV, for two baryon densities.

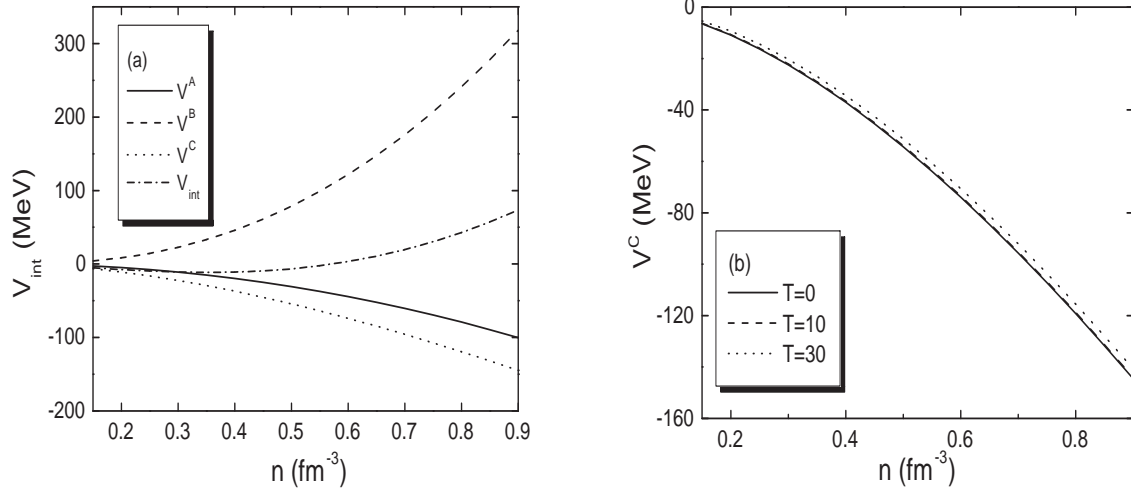


Figure 2: a) The contribution of the various terms  $V^A$ ,  $V^B$ ,  $V^C$  and the total potential energy density  $V_{\text{int}}$  as a function of the baryon density b) The momentum dependent term  $V^C$  as a function of the baryon density at temperature  $T = 0, 10, 30$  MeV.

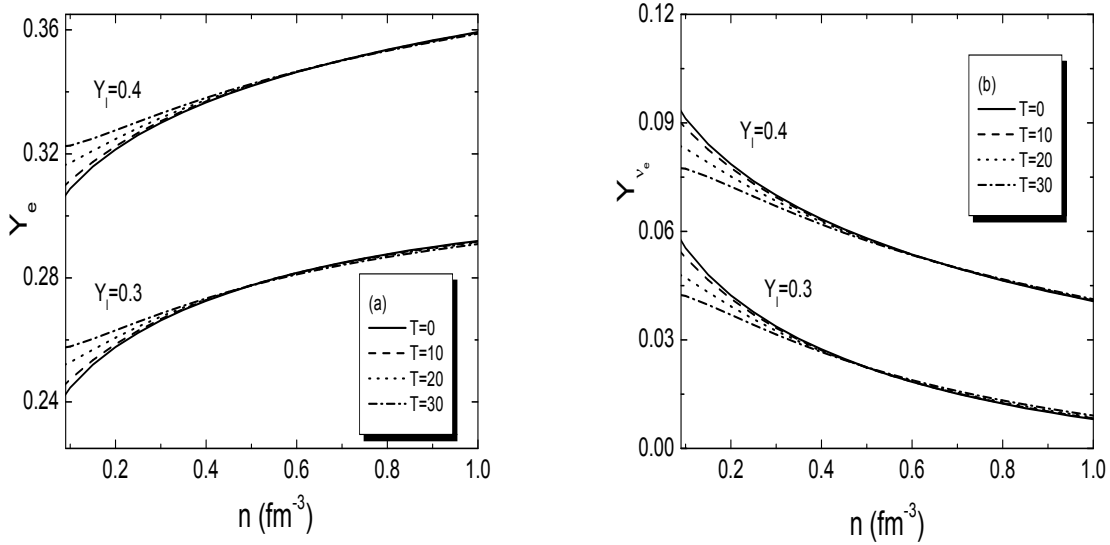


Figure 3: Electron  $Y_e$  and neutrino  $Y_{\nu_e}$  fraction versus baryon density for various values of  $T$  for total lepton fraction a)  $Y_l = 0.3$  and b)  $Y_l = 0.4$ .



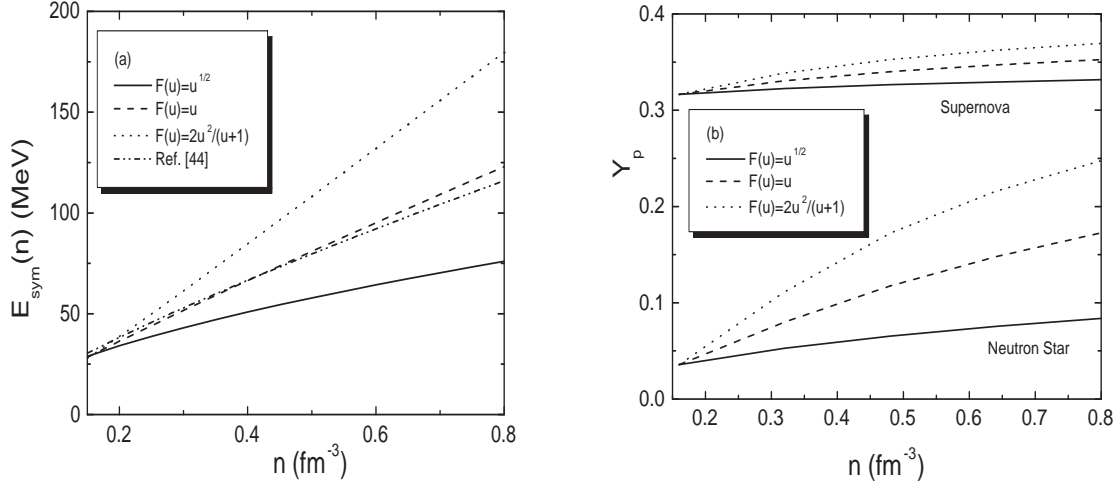


Figure 4: a) The nuclear symmetry energy for three different parametrization (equation (11)) of the interaction part with the results of reference [44] (see text for more details) and b) the proton fraction  $Y_p$  versus baryon density for cold neutron star matter (down curves) and supernova matter (up curves) for the three different parametrization of the nuclear symmetry energy.

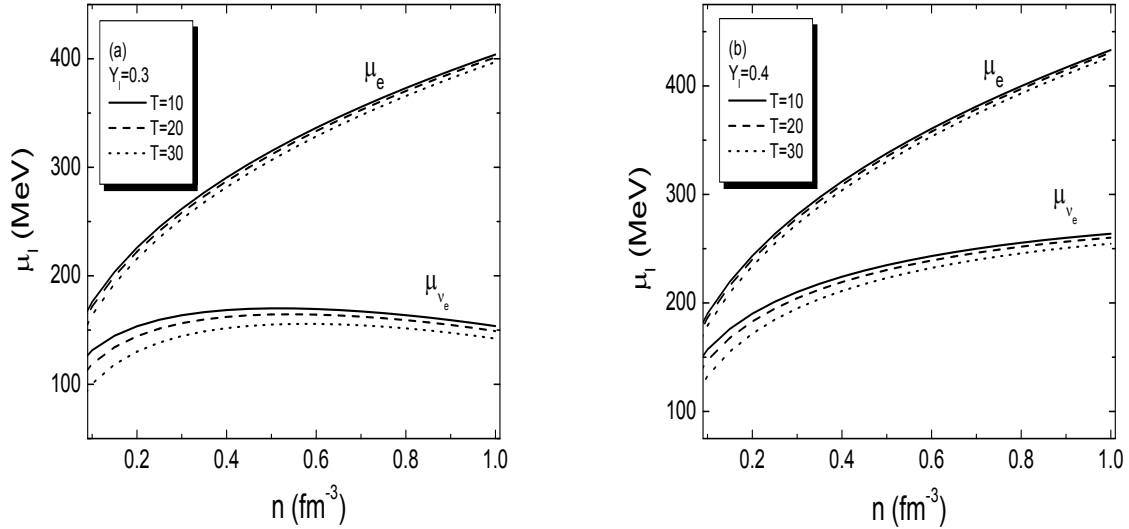


Figure 5: Electron  $\mu_e$  and neutrino  $\mu_{\nu_e}$  chemical potentials versus baryon density for various values of  $T$  for total lepton fraction a)  $Y_l = 0.3$  and b)  $Y_l = 0.4$ .

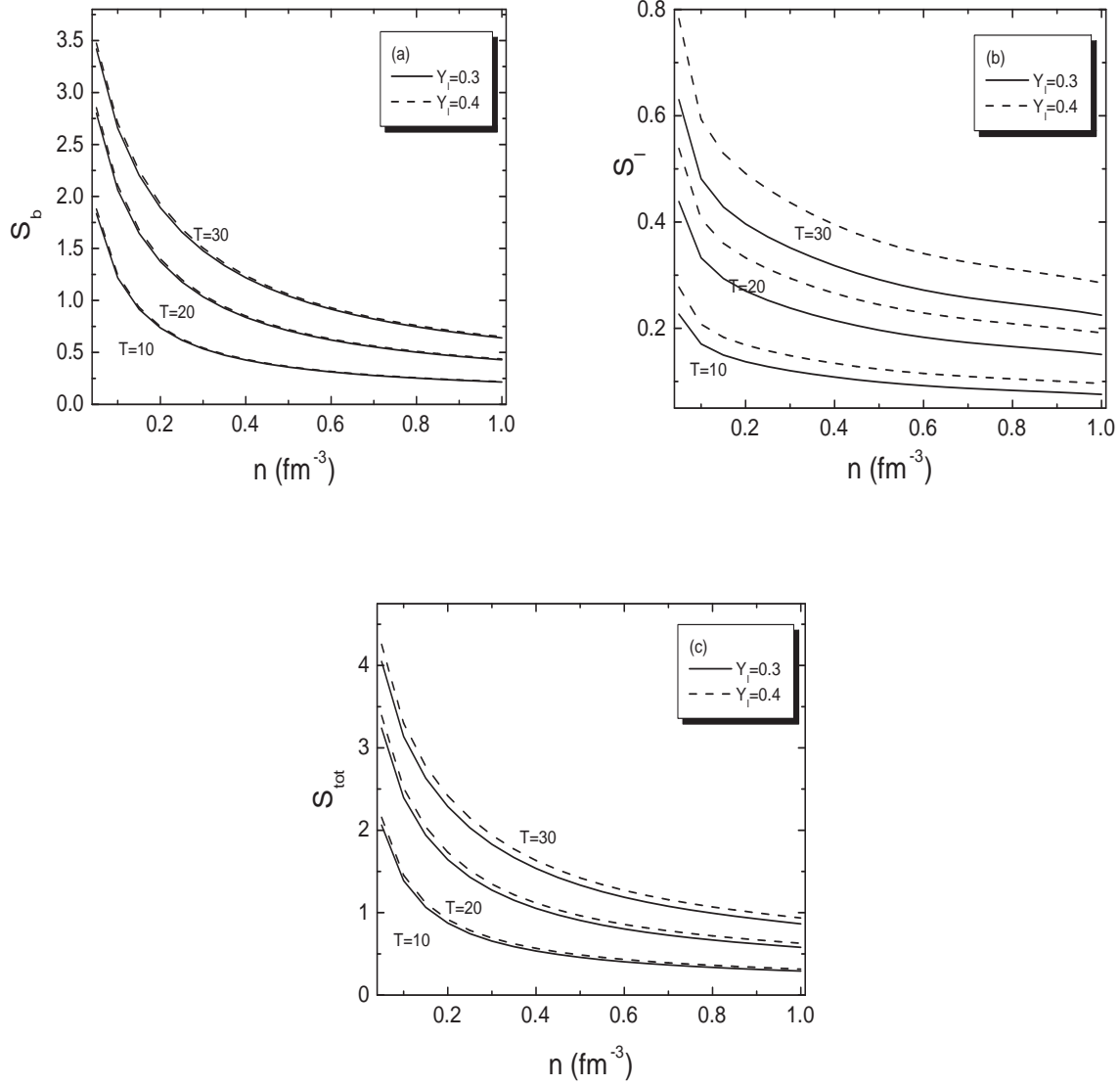


Figure 6: Contribution to the total entropy per particle of a) baryons ( $S_b$ ), b) leptons ( $S_l$ ) and c) the total entropy ( $S_{tot}$ ) versus the baryon density for various values of  $T$  for total lepton fraction  $Y_l = 0.3$  and  $Y_l = 0.4$ .

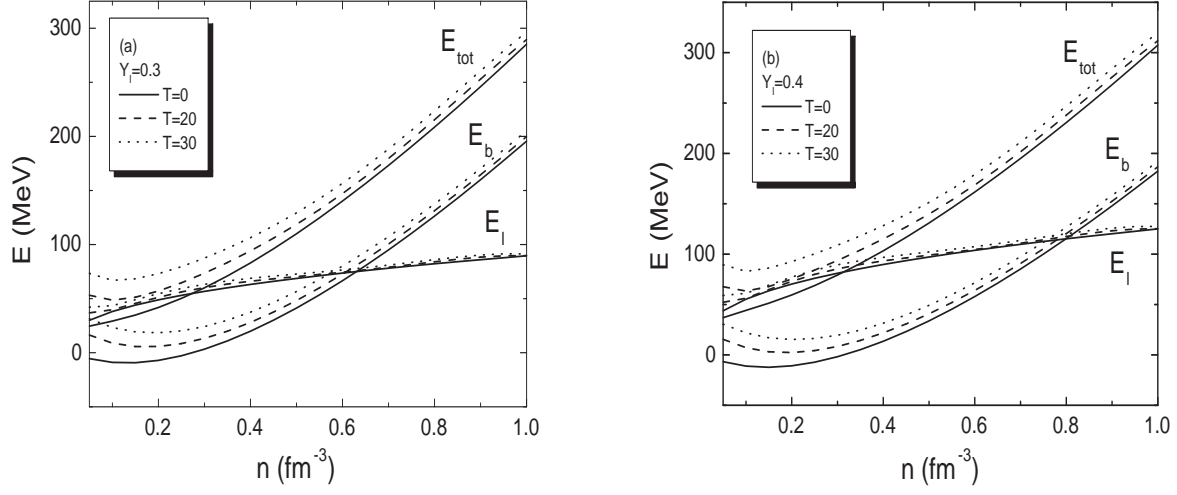


Figure 7: Contribution to the total energy per particle of baryons ( $E_b$ ), leptons ( $E_l$ ) and the total energy ( $E_{\text{tot}}$ ) versus the baryon density for various values of  $T$  for total lepton fraction a)  $Y_l = 0.3$  and b)  $Y_l = 0.4$ .

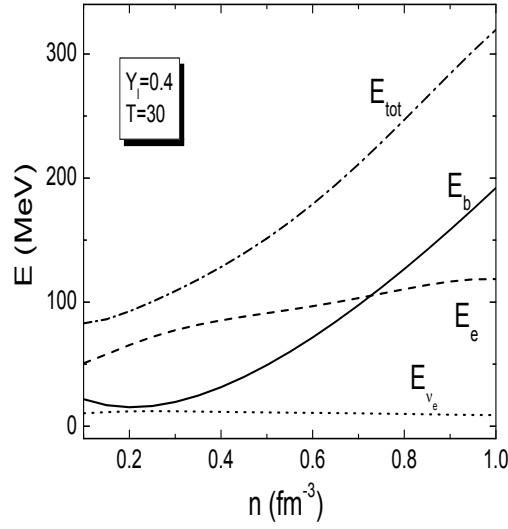


Figure 8: Energies per particle of respective components  $E_i$  versus the baryon density  $n$  for the specific case with  $T = 30$  MeV and  $Y_l = 0.4$ .

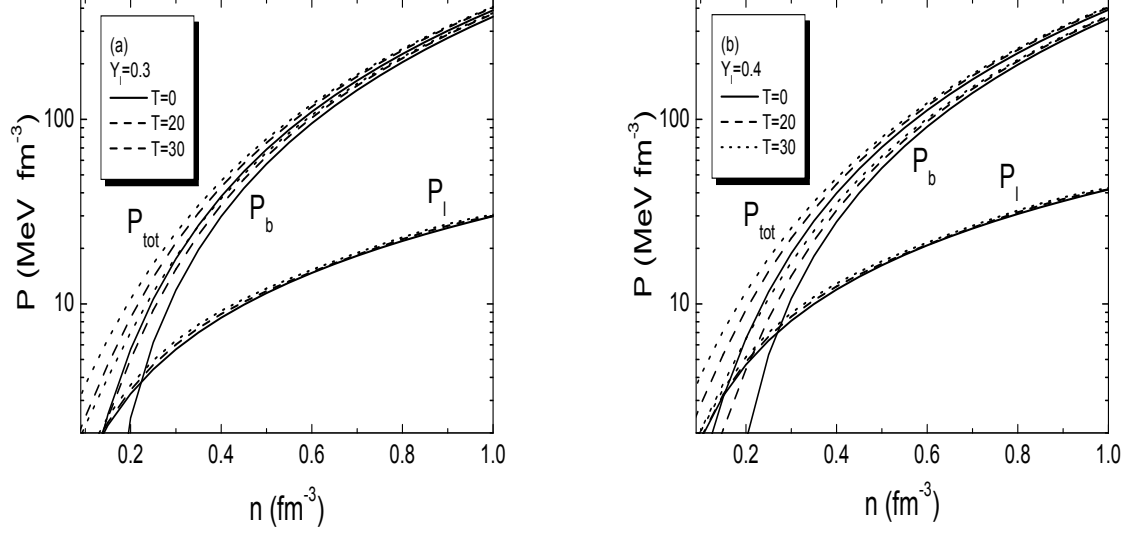


Figure 9: The pressure of baryons ( $P_b$ ), leptons ( $P_l$ ) and the total pressure ( $P_{\text{tot}}$ ) versus the baryon density for various values of  $T$  for the cases a)  $Y_l = 0.3$  and b)  $Y_l = 0.4$ .

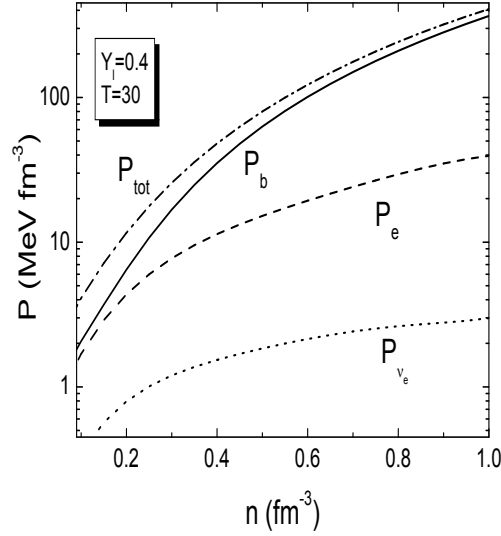


Figure 10: Pressures of respective components  $P_i$  versus the baryon density  $n$  for the specific case with  $T = 30$  MeV and  $Y_l = 0.4$ .

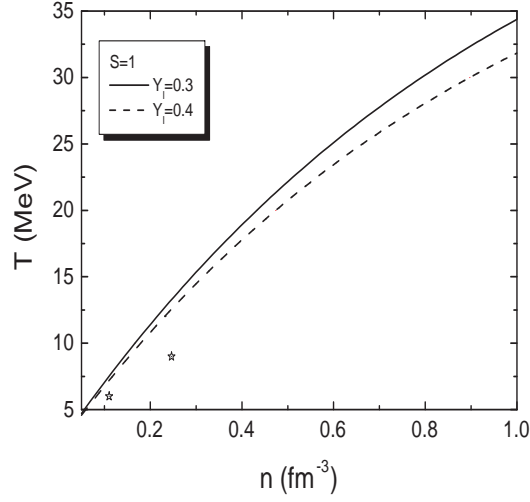


Figure 11: Temperature  $T$ -density  $n$  relation with  $Y_l = 0.3$  (solid line) and  $Y_l = 0.4$  (dashed line) for  $S = 1$ . Stars denote the results for the case with  $S = 1$  and  $Y_l = 0.4$ , extracted from the results by Lattimer et al. [19].

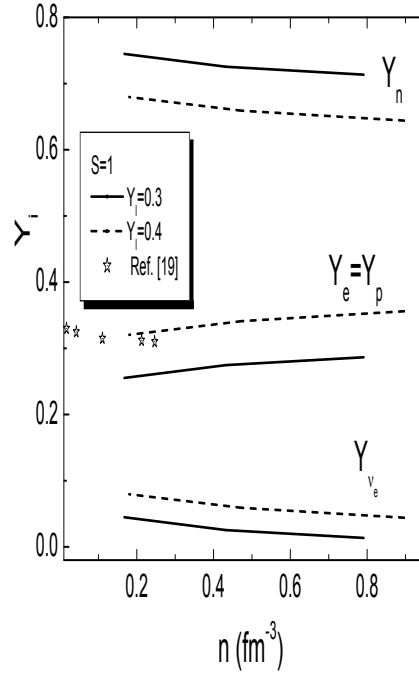


Figure 12: Fractions of respective components  $Y_i$  as functions of the density  $n$  with  $S = 1$ , for  $Y_l = 0.3$  (solid lines) and  $Y_l = 0.4$  (dashed lines). Stars denote the results of  $Y_p$  for the case with  $S = 1$  and  $Y_l = 0.4$ , extracted from the results by Lattimer et al. [19].

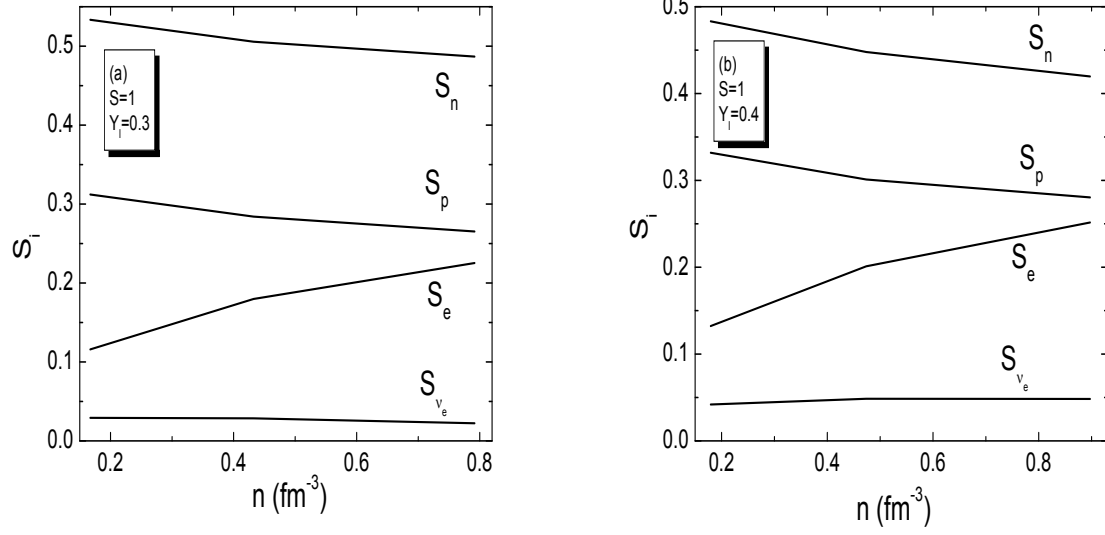


Figure 13: Entropies per baryon of respective components  $S_i$  contributing to the total entropy  $S = 1$  of supernova matter with total lepton fraction a)  $Y_l = 0.3$  and b)  $Y_l = 0.4$ , as functions of the density  $n$ .

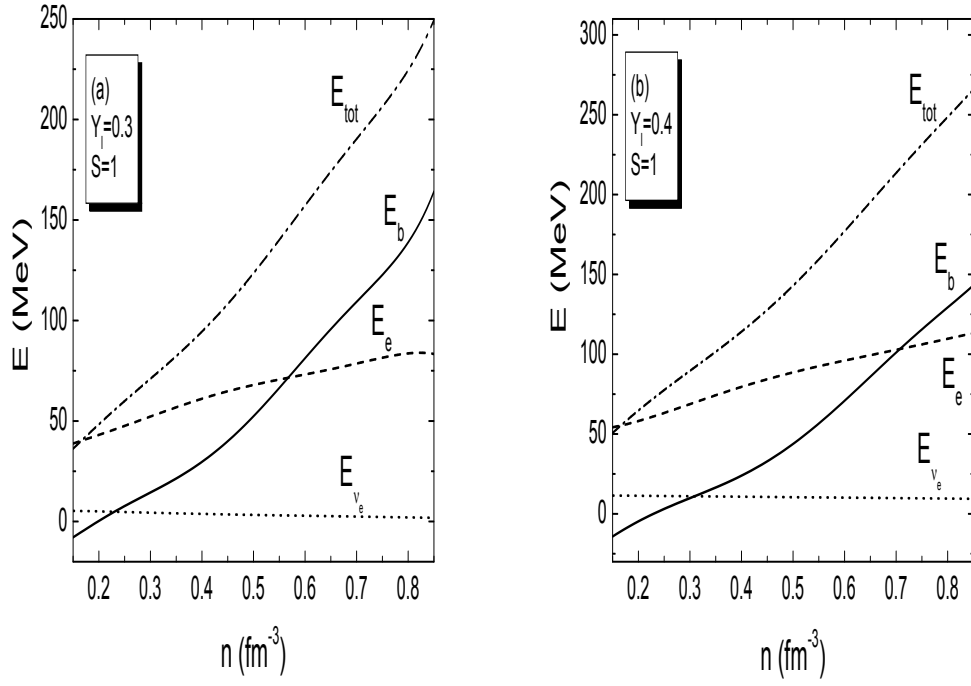


Figure 14: Internal energies per baryon of respective components  $E_i$  versus  $n$  with  $S = 1$  for the cases a)  $Y_l = 0.3$  and b)  $Y_l = 0.4$ .

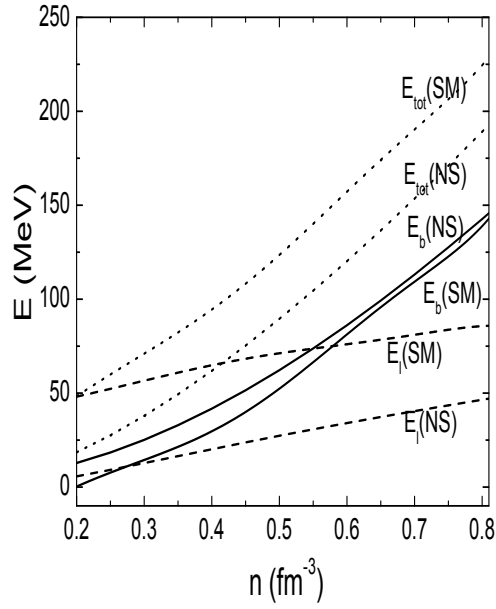


Figure 15: Internal energy per baryon versus density  $n$  for dense supernova matter (SM) in comparison with that of cold neutron star matter (NS) by applying the same model. The case of supernova matter corresponds to  $S = 1$  and  $Y_l = 0.3$ . The contribution of each species is plotted separately.